1. Test of Austin's Custom LaTeX Style

This document tests various custom commands and environments from austin.sty.

1.1. Custom Math Commands.

Here are some custom math symbols:

- Complex numbers: $\mathbb{C}$
- Natural numbers: $\mathbb{N}$
- Rational numbers: $\mathbb{Q}$
- Real numbers: $\mathbb{R}$
- Integers: $\mathbb{Z}$

Some custom operators: $\operatorname{cis}(\theta) = \cos(\theta) + i\sin(\theta)$
The least common multiple: $\operatorname{lcm}(12, 18) = 36$
Automorphism group: $\operatorname{Aut}(G)$

1.2. Custom Math Shortcuts.

Cube root: $\sqrt[3]{27} = 3$
Floor and ceiling: $\lfloor 3.7 \rfloor = 3$ and $\lceil 3.2 \rceil = 4$
Boxed result: $\boxed{E = mc^2}$
Fraction shortcut: $\frac{a}{b} = \frac{a}{b}$
Vector notation: $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

1.3. Theorem-Like Environments.

Theorem 1.1 (Fundamental Theorem of Calculus):

If f is continuous on $[a, b]$ and F is an antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Lemma 1.2 (Zorn's Lemma):

Every partially ordered set in which every chain has an upper bound contains at least one maximal element.

Corollary 1.3:

Every vector space has a basis.

Proposition 1.4 (Cauchy-Schwarz Inequality):

For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$:

$$|\langle \mathbf{u}, \mathbf{v} \rangle| \leq \|\mathbf{u}\| \|\mathbf{v}\|$$

Definition 1.5 (Group):

A group is a set G together with a binary operation $\cdot : G \times G \to G$ satisfying:

(1) Associativity: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
(2) Identity: $\exists e \in G$ s.t. $a \cdot e = e \cdot a = a$ for all $a \in G$
(3) Inverses: For each $a \in G$, $\exists a^{-1} \in G$ s.t. $a \cdot a^{-1} = a^{-1} \cdot a = e$

Example 1.6 (Symmetric Group):

The symmetric group S_n consists of all permutations of n elements. It has order $n!$.

Remark 1.7:

The notation $\text{Hom}(G, H)$ denotes the set of all homomorphisms from G to H.

1.4. **Probability and Statistics.** Expected value: $\mathbb{E}[X] = \sum_x x \cdot P(X = x)$
Variance: $\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$
Covariance: $\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$
Bernoulli distribution: $X \sim \text{Bern}(p)$
Binomial distribution: $Y \sim \text{Binom}(n, p)$

```
def fibonacci(n):
    if n <= 1:
        return n
    return fibonacci(n-1) + fibonacci(n-2)
```

1.6. **Problem Environment.**

Problem 1:

Prove that $\sqrt{2}$ is irrational.

a:

Assume $\sqrt{2} = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $\gcd(p, q) = 1$.

b:

Derive a contradiction.